# Biblical Pi

By **Allison Tawil**

As the Torah is an all-encompassing holy work, it contains all past, present, and future histories. It is therefore not surprising that material can be found in the Torah centuries before it is "discovered." Such is the case with pi. While there are skeptics who believe that the information found in the Torah lagged behind the knowledge of mathematicians who studied pi, others find hints within the Torah that prove an awareness to the value of pi long before mathematicians had a firm grip on its value. This latter view would give further proof of the Torah's endless knowledge.

Shlomo *HaMelech* explains in *Kohelet* (7:29) that, "Gd has made man upright; but they have sought out many schemes." Man is programmed to think straight, but nature is not built that way. This idea is verified by the Sages of the Talmud Yerushalmi who tell us, "there are no squares in nature" (*Nedarim* 3:2). This statement is true with the exception of crystals, which have straight edges in the shape of a regular polygon. Besides for such crystals, objects in nature cannot be precisely quantified or measured. While humans try to capture nature's essence, nature can never be fully understood by man [1].

Pi ($\pi$) is the Greek letter that represents the ratio between a circle's diameter (d), its length from one end to the other passing through its center, and its circumference (C), the circle's total outer perimeter. This formula is represented by $C=\pi a$. The area (A) of a circle is also found using the value of pi; area is equal to the radius (r) of the circle, or half of its diameter, squared, multiplied by pi. This formula is represented by $A=\pi r^2$ [2]. And true to the plight of humans, pi helps to quantify and measure the curves of nature, attempting to make nature somewhat more understandable. However, true to the ways of nature, this task is impossible because pi has an infinite number of decimal places. Pi's value starts off with 3.14159, but cannot be quantified completely since it goes on forever [1].

Pi's history is a long and interesting one. It begins with the ancient Babylonians around the year 1900 BCE, who approximated pi's value at 3.125. The ancient Egyptians estimated its value to be 3.1605. By the third century BCE, we had a fairly precise calculation of pi, offered by Archimedes of Syracuse, one of the greatest mathematicians of the ancient world. He calculated that $\frac{223}{71} < \pi < \frac{22}{7}$ ($3.1408 < \pi < 3.1429$), based on inscribing and circumscribing regular polygons into a circle. He was able to easily calculate the area of the two polygons, and knew that the area of the circle was a value between the areas of the two polygons (Figure 1) [2, 3].

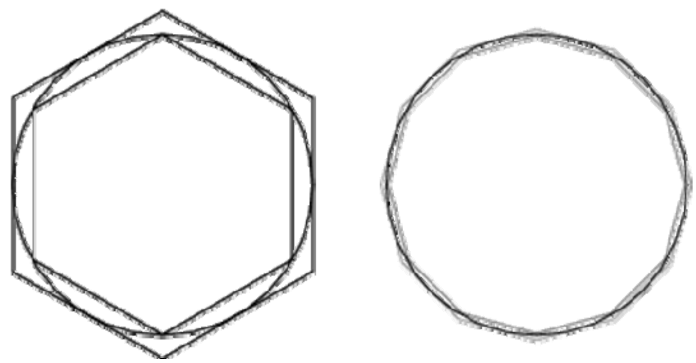

***Figure 1.*** *Archimedes' inscribed and circumscribed polygons allowed him to easily find the area of the inner circle.*

Over time, the approximations of pi became increasingly accurate, and in 2011, 10 trillion decimal places of pi were quantified [4].

The first mention of circular dimensions within the Torah is found during the construction of the *Bet HaMikdash*, when Shlomo *HaMelech* built "a molten sea, ten *amot* (cubits) from one brim to the other; it was round all around, and its height was five *amot*; and a line of thirty *amot* measured the circle around it" (*Melachim* I, 7:23). There is a nearly identical passage found later in Tanach, "Also he made a molten sea of ten *amot* from brim to brim, it was round, and five *amot* its height; and a line of thirty *amot* encircled it" (*Divrei HaYamim* II, 4:2). Based on these passages, one can assume that the understood ratio of diameter to circumference of a circle used during that time (around 550 BCE) was three. The problem is that by 1650 BCE, over 1000 years prior to the construction of the *Bet HaMikdash*, there was a more accurate value of pi being used. If this closer approximation of pi was being used by Shlomo and the diameter of this sea was 10 cubits, then the circumference of the sea would have been: $C=\pi d \rightarrow 10$ cubits $\times 3.14 = 31.4$ cubits. But the *pasuk* relates that the circumference of the sea was 30 cubits, not 31, which would have been the correct rounded value in this case.

The Talmud also presents the ratio between a circle's diameter and circumference to be three. In the Talmud Bavli (*Eruvin* 14a), Rabbi Yohanan quotes the *pasuk* in

*Melachim* I to prove that the ratio is three. Rabbi Yohanan did not say that one should use the number three as pi because three is pi's exact value, but rather because three is the value given in the Torah [5]. To further validate the Torah's use of this value, the Gemara questions this *pasuk* and asks about the thickness of the brim of the pool. Rav Papa replies that the width was very thin, like a flower petal. However, the Gemara notes that even a thin width would be a fraction to be included in the measurements. The Gemara therefore concludes that the measurements were taken from the inside of the brim, and the measurement stated in the *pasuk* is the pool's inner circumference. Based on this passage, Tosafot gathers that the Sages of the Gemara took the calculation of 30 cubits to be exact; otherwise they would not have asked about the width of the brim. Tosafot concludes that this is not an accurate calculation based on actual mathematics.

This ratio of three is repeated again in the Talmud Bavli (*Baba Batra* 14b), where the rule is given that "anything which has in its circumference three *tefachim* (handbreadths) has in its diameter a *tefach*." The Talmud then discusses this rule in the case of a Torah scroll. Assuming that the circumference of a Torah scroll is 6 *tefachim*, based on the aforementioned rule, the scroll's diameter would be 2 *tefachim* ($C=\pi d \rightarrow 6$ *tefachim* $= 3 \times 2$ *tefachim*). It is stated that the width inside the *Aron Kodesh*, the Holy Ark, is 2 *tefachim*. Since this is the case, the Gemara questions how such a scroll would fit into the *Aron*. The Sages give a detailed explanation of how to fold the scroll to fit it in and conclude that it would fit into the *Aron*, but with great difficulty. However, if they would have used a more accurate approximation of pi, the scroll's diameter would have been found to be less than two *tefachim* (6 *tefachim* $\div$ 3.14 = ~1.9 *tefachim*), and there would be no problem fitting the scroll into the *Aron*. This Gemara gives proof to skeptics who claim that the Sages of the Talmud lagged behind the mathematicians of their time in the calculation of pi [6]. Nevertheless, the Rambam, 12$^{th}$ century philosopher, astronomer, Torah scholar, and physician, explains how the value of three is not an error at all. Since pi is an irrational number that goes on forever, *Chazal* decided to round off the number to simplify calculations that require the use of pi. There could never be more than an approximation of the value of pi because it is endless, and thus it is simply a choice of how many decimal places one approximates.

Others suggest that there may be practical reasoning as to why the Torah uses three for pi instead of a more accurate ratio, such as 3.1. For example, a value of three instead of a slightly higher (and more accurate) value would protect a customer from being overcharged when buying round matzah, based on complicated mathematical calculations. It is also possible that the value of three was used for everyday calculations to simplify transactions and measurements.

Rabbi Haim David Z. Margaliot, in his 1938 work *Dover Yesharim*, explains otherwise. He says that Tanach uses the ancient means of measurement to find the circumference of a circle. The circumference used to be understood as the measurement of a polygon inscribed inside of a circle. He suggests that the rabbinical use of the value three for pi is justified given this means of measuring a circumference. By using a stick the length of the radius of a given circle, they would count how many times this stick would need to go around to complete the circle (Figure 2). It took six times to go around, so the circle's circumference was six times its radius, or three times its diameter [7, 8].

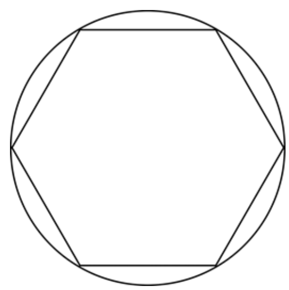

***Figure 2.***

*Hexagon in the circle made up of sides the size of the circle's radius*

The Talmud presents other cases in which the ratio of three is used for pi. Rabbi Shimon ben Tsemah (the Tashbetz), fifteenth century *posek* and student of mathematics, astronomy, philosophy, and medicine, finds evidence in one of the Talmud's discussions that the Sages may have indeed had a more precise value for pi.

The Talmud Bavli (*Sukkah* 7b-8a) gives its take on the age-old geometry problem called "squaring the circle" (Figure 3). The problem of squaring the circle attempts to find a square that has the same area as a circle. Because pi is an irrational number, there will never be an exact value for a diameter that will give a circle the same area as a certain square. In this Talmudic passage, the Sages try to find the size of a valid circular *sukkah*.

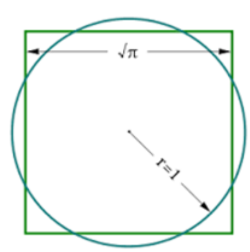


***Figure 3.***

*The ancient problem of squaring a circle*

According to Rav, a valid square *sukkah* should be a minimum of four cubits by four cubits. Based on Rav's dimensions for a square *sukkah*, Rabbi Yohanan declares that a circular *sukkah* should be large enough to fit at least twenty-four people around the circumference, assuming that each person takes up one cubit of space. Note that this does not mean that a *sukkah* is required to fit at least twenty-four people - this was just a method used to estimate circumference.

The Talmud proceeds to do the math of what minimum size a circular *sukkah* should be. After circumscribing a minimum sized square *sukkah*, it can be seen that the circular *sukkah*'s diameter is the square's diagonal (Figure 4).

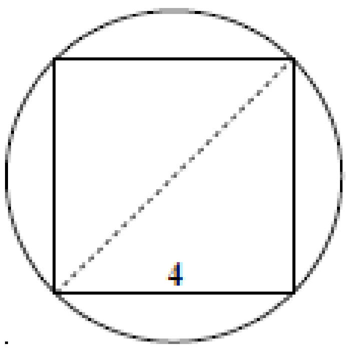


***Figure 4.***

*A minimum sized square sukkah circumscribed in a minimum size circular sukkah. The diameter of the circular sukkah is equal to the diagonal of the square sukkah.*

The Talmud has a rule to calculate the length of a square's diagonal - "each handbreadth in a square is $^7/_5$ handbreadths in its diagonal." Based on this rule, which is basically the approximation of $\sqrt{2}$, we find that the diagonal of such a minimum size square *sukkah* is equal to 4 cubits × $^7/_5$ = 5.6 cubits. (This value is very close to the actual diagonal of the square found by the Pythagorean Theorem, $4^2 + 4^2 = 32 \rightarrow \sqrt{32} = 5.66$). The circumference of the circle is its diameter (the diagonal) times the Talmud's value of pi → 5.6 cubits × 3 =16.8 cubits. Therefore, in reality, the minimum circumference of a circular *sukkah* should be 16.8 cubits, not twenty-four.

Rav Assi*,* trying to give a reason for Rabbi Yohanan's value of twenty-four cubits, suggests that Rabbi Yohanan meant that twenty-four people are able to sit around the outside of the *sukkah*. The circumference of the circle that circumscribes twenty-four people would be twenty-four cubits (Figure 5). The diameter of this circle would be its circumference divided by the Talmud's pi→ 24 cubits ÷ 3 = 8 cubits. The diameter of the actual *sukkah* would be 8 - 2 cubits (one cubit for each person on the outside) = 6 cubits. The circumference of this *sukkah* is 6 cubits × 3= 18 cubits. Although this value is closer to Rabbi Yohanan's, it still does not match. This leads Rav Assi to conclude that Rabbi Yohanan was being inexact in his approximations.

However, it is known that Rabbi Yohanan was a very precise person. He was even quoted as saying (*Shabbat*, 145b), "if it is clear as day, say it; if not, do not say it." There must have been a reason for Rabbi Yohanan to give the value of twenty-four cubits, especially given the fact that using Rav Assi's calculations, it would be a sufficient sized circular *sukkah* with twenty-three people surrounding the *sukkah* (as this would give a *sukkah* with a circumference of seventeen cubits, which is much closer to the Gemara's 16.8 cubits).

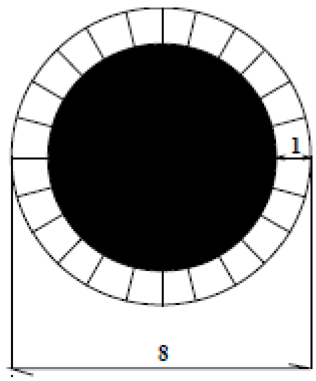


***Figure 5.***

*A minimum size circular sukkah surrounded by twenty-four people, each taking up one cubit of space.*

The Tashbetz therefore suggests that Rabbi Yohanan used more precise values of π and $\sqrt{2}$. Assuming that Rabbi Yohanan made π equal to $3^1/_7$ and $\sqrt{2}$ equal to a value a little greater than $1^2/_5$. The minimum circumference of the circle (taken from the circumscribed 4×4 *sukkah*) would be 4 cubits × $1^2/_5$ × $3^1/_7$ = a little greater than $17^3/_5$ cubits. The circumference of the *sukkah* would be (using the same math as before, but with more accurate numbers) $17^5/_7$. The difference between these two values is less than $^4/_{35}$ cubits. The Tashbetz's explanation suggests that approximations are used when teaching students, but more exact values are used by the experts, such as Rabbi Yohanan, when doing calculations [7, 9, 10].

There is also an explanation by the Vilna Gaon, 18th century Talmudist, *posek*, kabbalist, and mathematician. He notices a discrepancy in the *pasuk* in *Melachim I* that first introduces the idea of pi. The *pasuk* writes out the word "*kav*" (meaning circumference) as *kuf*, *vav*, *heh*. The *heh* at the end of the word is not pronounced, so the word is pronounced just as a *kuf vav*. Using *gematria* to give value to the numbers, *kuf* is 100, *vav* is 6, and *heh* is 5. The value of the spelled out word is 100+6+5= 111. The value of the pronounced word is 100 + 6= 106. When dividing the former by the latter, we get the ratio 111/106=1.0472.

When taking the ratio between a more precise value of π and the *pasuk*'s value for π we have 3.14159/3=1.0472. The Vilna Gaon does not take these matching values as a coincidence, but rather as a proof to the Torah's timelessness - it reveals a precise value of pi long before it was known to mathematicians [11].

Although there are those who see an erred value of pi when reading Tanach and the Talmud, there are many ways to see that the Torah is never faulted. It is an all encompassing work that contains all knowledge, past, present, and future.


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